Section 6.6 – Average and Marginal Product of Labour

Questions

1. A firm’s short-run production function is given by
   \[ Q = 180L^2 - 3L^3. \]
   Find the marginal and average products of labour.
   Check that when average product is maximized, APL=MPL.

2. The output a firm produces per week is given by its short run production function
   \[ Q = 91L + 16L^2 - L^3 \]
   where L is the number of persons employed.
   (a) Find the average product of labour and show at what number of employees it reaches a maximum.
   (b) Obtain an expression for the marginal product of labour. Over what range of number of employees do diminishing marginal returns occur?
   (c) What is the maximum value of the average product of labour? What is the value of the marginal product of labour at that point?

3. For a firm with the short-run production function
   \[ Q = 42L + 27L^2 - 0.5L^3 \]
   find APL and show at what number of employees it reaches a maximum.
   Find also MPL and show when there are diminishing marginal returns.
Answers

1. MPL = \(dQ/dL = 360L - 9L^2\)
   APL = \(Q/L = 180L - 3L^2\)
   \(d(APL)/dL = 180 - 6L = 0\) (for a maximum)
   So, \(L = 30\) is a turning point. The second derivative is
   \(d^2(APL)/dL^2 = -6\)
   The negative sign indicates a maximum, so APL is a maximum at \(L = 6\).
   Substituting in APL and MPL we find
   APL = 180(30) – 3(30^2) = 2700
   MPL = 360(30) – 9(30^2) = 2700
   APL and MPL intersect at the maximum value of APL.

2. (a) APL = 91 + 16L – L^2
   \(d(APL)/dL = 16 - 2L = 0\) (for a maximum)
   So, \(L = 8\) is a turning point. The second derivative is
   \(d^2(APL)/dL^2 = -2\)
   This is negative, so \(L = 8\) is a maximum.
   MPL = 91 + 32L – 3L^2
   \(d(MPL)/dL = 32 - 6L = 0\) (for a maximum)
   \(L = 5.33\) is a turning point. The second derivative is
   \(d^2(MPL)/dL^2 = -6\)
   The negative value indicates a maximum.
   After the maximum MPL falls, or diminishes, so diminishing marginal returns occur when more than 5.33 people are employed (or when 6 or more people are employed if all employees work full time).
   (c) Substitute \(L = 8\)
   APL = 91 + 16(8) – (8^2) = 155
   MPL = 91 + 32(8) – 3(8^2) = 155 = APL
3. \[ APL = 42 + 27L - 0.5L^2 \]
\[ \frac{d(APL)}{dL} = 27 - L = 0 \text{ (for a maximum)} \]
So, \( L = 27 \) is a turning point. The second derivative is
\[ \frac{d^2(APL)}{dL^2} = -1 \]
This is negative, so \( L = 27 \) is a maximum.

\[ MPL = 42 + 54L - 1.5L^2 \]
\[ \frac{d(MPL)}{dL} = 54 - 3L = 0 \text{ (for a maximum)} \]
So, \( L = 18 \) is a turning point. The second derivative is
\[ \frac{d^2(MPL)}{dL^2} = -3 \]
This is negative, so \( L = 18 \) is a maximum.
Diminishing marginal returns occur when more than 18 people are employed.