**TIME VALUE OF MONEY OR DISCOUNTED CASH FLOW ANALYSIS**

- Very Important concept -- Almost everything from this point on in finance based upon understanding this concept.

- You **must** get this material down “cold.”

- You can do problems using formulas, calculators, and spreadsheets. We will primarily use financial calculators. Please have your financial calculators with you in class.

---

**Types of Interest**

- **Simple Interest**
  
  Interest paid (earned) on only the original amount, or principal borrowed (lent).

- **Compound Interest**
  
  Interest paid (earned) on previous interest earned, as well as on the principal borrowed (lent).

---

**The Concept of TVM is One of Finance’s BIG contributions:**

- A valid way for valuing different patterns of cash flows on a common basis.
- A way of cutting through exaggerated claims.
- The easiest way to envision this is using Time Lines:

---

**The Time Value of Money**

**Compounding and Discounting Single Sums**
Why TIME

Why is TIME such an important element in your decision?
TIME allows you the opportunity to postpone consumption and earn INTEREST.

We know that receiving $1 today is worth more than $1 in the future. This is due to opportunity costs. The opportunity cost of receiving $1 in the future is the interest we could have earned if we had received the $1 sooner.

If we can measure this opportunity cost, we can:

- Translate $1 today into its equivalent in the future (compounding).
- Translate $1 in the future into its equivalent today (discounting).

If we can measure this opportunity cost, we can:

Today Future

If we can measure this opportunity cost, we can:

Today Future

Today Future
If we can measure this opportunity cost, we can:

• Translate $1 today into its equivalent in the future (compounding).

Today  Future
?

• Translate $1 in the future into its equivalent today (discounting).

Today Future
?

Look carefully at the timeline:

• The timeline allows you to
  - visualize the actual situation.
  - include all elements (n,i,FV,PV,+S,-S)
  - understand the dynamics.
  - make accurate calculations, every time.
  - speed up the process of problem solving.

Why use Timelines?

- *useful* for accurate TVM calculations.
- *essential* for capital budgeting.
- *essential* for understanding TVM.
- *essential* for incremental cash flows.
- *essential* for succeeding in Finance.

Future Value - single sums

If you deposit $100 in an account earning 6%, how much would you have in the account after 1 year?

\[
FV = \text{Future Value} = \text{single sums}
\]

Future Value - single sums

If you deposit $100 in an account earning 6%, how much would you have in the account after 1 year?

\[
PV = \text{Present Value} = FV = \text{Future Value}
\]
If you deposit $100 in an account earning 6%, how much would you have in the account after 1 year?

**Calculator Solution:**
P/Y = 1  
I = 6  
N = 1  
PV = -100  
FV = $106

**Mathematical Solution:**  
\[ FV = PV \times (FVIF_{i, n}) \]  
\[ FV = 100 \times (FVIF_{.06, 1}) \]  
\[ FV = PV \times (1 + i)^n \]  
\[ FV = 100 \times (1.06)^1 = $106 \]

If you deposit $100 in an account earning 6%, how much would you have in the account after 5 years?

**Calculator Solution:**
P/Y = 1  
I = 6  
N = 5  
PV = -100  
FV = $133.82

**Mathematical Solution:**  
\[ FV = PV \times (FVIF_{i, n}) \]  
\[ FV = 100 \times (FVIF_{.06, 5}) \]  
\[ FV = PV \times (1 + i)^n \]  
\[ FV = 100 \times (1.06)^5 = $133.82 \]
Future Value - single sums

If you deposit $100 in an account earning 6%, how much would you have in the account after 5 years?

**Calculator Solution:**

\[ \text{PV} = -100 \quad \text{i} = 6 \quad \text{N} = 5 \quad \text{PV} = -100 \]

\[ \text{FV} = 133.82 \]

**Mathematical Solution:**

\[ \text{FV} = \text{PV} \times (\text{FVIF}_i^n) \]

\[ \text{FV} = 100 \times (\text{FVIF}_{0.06}^5) \text{ (use FVIF table, or)} \]

\[ \text{FV} = 100 \times (1 + 0.06)^5 \]

\[ \text{FV} = 133.82 \]

Future Value - single sums

If you deposit $100 in an account earning 6% with quarterly compounding, how much would you have in the account after 5 years?

**Calculator Solution:**

\[ \text{P/Y} = 4 \quad \text{i} = 6 \quad \text{N} = 20 \quad \text{PV} = -100 \]

\[ \text{FV} = 134.68 \]

**Mathematical Solution:**

\[ \text{FV} = \text{PV} \times (\text{FVIF}_i^n) \]

\[ \text{FV} = 100 \times (\text{FVIF}_{0.06/4}^{20}) \text{ (use FVIF table, or)} \]

\[ \text{FV} = 100 \times (1.015)^{20} \]

\[ \text{FV} = 134.68 \]
Mathematical Solution:

\[ FV = PV \times (FVIF_{i,n}) \]
\[ FV = 100 \times (FVIF_{0.015, 20}) \]  
(can’t use FVIF table)

\[ FV = PV \times (1 + \frac{i}{m})^{m \times n} \]
\[ FV = 100 \times (1.015)^{20} = \$134.68 \]

Future Value - single sums
If you deposit $100 in an account earning 6% with quarterly compounding, how much would you have in the account after 5 years?

\[ PV = -100 \]
\[ FV = 134.68 \]

Future Value - single sums
If you deposit $100 in an account earning 6% with monthly compounding, how much would you have in the account after 5 years?

Calculation Solution:

\[ P/Y = 12 \quad I = 6 \]
\[ N = 60 \quad PV = -100 \]

\[ FV = \$134.89 \]

Future Value - single sums
If you deposit $100 in an account earning 6% with monthly compounding, how much would you have in the account after 5 years?

\[ PV = -100 \]
\[ FV = 134.89 \]

Future Value - single sums
If you deposit $100 in an account earning 6% with monthly compounding, how much would you have in the account after 5 years?

Mathematical Solution:

\[ FV = PV \times (FVIF_{i,n}) \]
\[ FV = 100 \times (FVIF_{0.005, 60}) \]  
(can’t use FVIF table)

\[ FV = PV \times (1 + \frac{i}{m})^{m \times n} \]
\[ FV = 100 \times (1.005)^{60} = \$134.89 \]
**The Future Value of a Single Amount**

**Graphical Presentation**

**Different Interest Rates**

---

**Future Value - continuous compounding**

*What is the FV of $1,000 earning 8% with continuous compounding, after 100 years?*

**Mathematical Solution:**

\[ FV = PV \left( e^{\text{in}} \right) \]

\[ FV = 1000 \left( e^{0.08 \times 100} \right) = 1000 \left( e^{8} \right) \]

\[ FV = $2,980,957.99 \]

---

**Future Value - continuous compounding**

*What is the FV of $1,000 earning 8% with continuous compounding, after 100 years?*

**Mathematical Solution:**

\[ FV = PV \left( e^{\text{in}} \right) \]

\[ FV = 1000 \left( e^{0.08 \times 100} \right) = 1000 \left( e^{8} \right) \]

\[ FV = $2,980,957.99 \]
Compounding Periods

Frequency with which interest is credited for calculating future interest, usually annually, semiannually, quarterly, or monthly.

The shorter the period, the more interest is earned on interest:

- **Annually**
  - 12%
  - $100 \rightarrow $112

- **Semiannually**
  - 6%
  - $100 \rightarrow $106
  - $106 \rightarrow $112.36

- **Quarterly**
  - 3%
  - $100 \rightarrow $103
  - $103 \rightarrow $106.09
  - $106.09 \rightarrow $109.27
  - $109.27 \rightarrow $112.55

Quote the annual (nominal) rate ($k_{nom}$) stating the compounding period immediately afterward: 

- "12% compounded quarterly"

The Effective Annual Rate (EAR)

The rate of annually compounded interest equivalent to the nominal rate compounded more frequently.

**Table 5-2 Year End Balances at Various Compounding Periods $100 Initial Deposit and $k_{nom} = 12%**

<table>
<thead>
<tr>
<th>Compounding</th>
<th>Final balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual</td>
<td>$112.00</td>
</tr>
<tr>
<td>Semiannual</td>
<td>$112.36</td>
</tr>
<tr>
<td>Quarterly</td>
<td>$112.55</td>
</tr>
<tr>
<td>Monthly</td>
<td>$112.68</td>
</tr>
</tbody>
</table>

In general:

$$\text{EAR} = \left(1 + \frac{k_{nom}}{m}\right)^m - 1$$

Present Value - single sums

If you receive $100 one year from now, what is the PV of that $100 if your opportunity cost is 6%?

PV = 0
FV = 100

**Calculator Solution:**

P/Y = 1
I = 6
N = 1
FV = 100
PV = -94.34
### Present Value - single sums

If you receive $100 one year from now, what is the PV of that $100 if your opportunity cost is 6%?

**Calculator Solution:**

\[
PV = -94.34 \quad \text{FV} = 100
\]

\[
PV = \text{PV} = -94.34 \quad \text{FV} = 100
\]

**Mathematical Solution:**

\[
PV = FV \times (PVIF_{0.06, 1}) \quad \text{(use PVIF table, or)}
\]

\[
PV = \frac{FV}{(1 + i)^n}
\]

\[
PV = \frac{100}{1.06^1} = 94.34
\]

### Present Value - single sums

If you receive $100 five years from now, what is the PV of that $100 if your opportunity cost is 6%?

**Calculator Solution:**

\[
PV = -74.73 \quad \text{FV} = 100
\]

**Mathematical Solution:**

\[
PV = FV \times (PVIF_{0.06, 5}) \quad \text{(use PVIF table, or)}
\]

\[
PV = \frac{FV}{(1 + i)^n}
\]

\[
PV = \frac{100}{1.06^5} = 74.73
\]
**Present Value - single sums**

If you receive $100 five years from now, what is the PV of that $100 if your opportunity cost is 6%?

Mathematical Solution:

\[
PV = FV \times (PVIF_{i,n})
\]

\[
PV = 100 \times (PVIF_{0.06,5})
\]

(Use PVIF table, or)

\[
PV = \frac{FV}{(1 + i)^n}
\]

\[
PV = \frac{100}{1.06^5} = \$74.73
\]

**Present Value - single sums**

What is the PV of $1,000 to be received 15 years from now if your opportunity cost is 7%?

Calculator Solution:

\[
P/Y = 1 \quad I = 7
\]

\[
N = 15 \quad FV = 1,000
\]

\[
PV = -362.45
\]

**Present Value - single sums**

What is the PV of $1,000 to be received 15 years from now if your opportunity cost is 7%?

Mathematical Solution:

\[
PV = FV \times (PVIF_{i,n})
\]

\[
PV = 100 \times (PVIF_{0.07,15})
\]

(Use PVIF table, or)

\[
PV = \frac{FV}{(1 + i)^n}
\]

\[
PV = \frac{100}{1.07^{15}} = \$362.45
\]
If you sold land for $11,933 that you bought 5 years ago for $5,000, what is your annual rate of return?

Mathematical Solution:

\[ PV = FV \left( \frac{1}{1 + i} \right)^n \]
\[ 5,000 = 11,933 \left( \frac{1}{1 + i} \right)^5 \]
\[ .419 = \left( \frac{1}{1 + i} \right)^5 \]
\[ 2.3866 = (1+i)^5 \]
\[ (2.3866)^{1/5} = (1+i) \]
\[ i = .19\%

Suppose you placed $100 in an account that pays 9.6% interest, compounded monthly. How long will it take for your account to grow to $500?

Calculator Solution:

- P/Y = 12
- FV = 500
- I = 9.6
- N = 202 months
Suppose you placed $100 in an account that pays 9.6% interest, compounded monthly. How long will it take for your account to grow to $500?

**Mathematical Solution:**

\[ PV = \frac{FV}{(1 + i)^n} \]

\[ 100 = \frac{500}{(1 + 0.008)^n} \]

\[ 5 = (1.008)^n \]

\[ \ln 5 = \ln (1.008)^n \]

\[ \ln 5 = n \ln (1.008) \]

\[ 1.60944 = 0.007968 N \]

\[ N = 202 \text{ months} \]

**Hint for single sum problems:**
- In every single sum future value and present value problem, there are 4 variables:
- FV, PV, i, and n
- When doing problems, you will be given 3 of these variables and asked to solve for the 4th variable.
- Keeping this in mind makes “time value” problems much easier!

**Present Value - An Example**

Suppose you have the opportunity to buy a piece of land for $10,000 today, and sell it in eight years for $20,000. Is this a “good deal” if you can put your money in a risk-equivalent that is expected to earn 10 percent a year compounded annually?

**Facts**

**Timeline**

**Present Value - An Example**

The present value of the $20,000 you expect to receive at the end of eight years is:

\[ PV = \frac{20,000}{(1.10)^8} = 9330.15 \]

This is a “bad deal” since the present value of return in eight years is less than the cost of the land.

**The Time Value of Money**

Compounding and Discounting

Cash Flow Streams

**Annuities**

- Annuity: a sequence of equal cash flows, occurring at the end of each period.
**Annuities**

• **Annuity:** a sequence of equal cash flows, occurring at the end of each period.

![Annuity Diagram](image)

**Examples of Annuities:**

• If you buy a bond, you will receive equal semi-annual coupon interest payments over the life of the bond.

• If you borrow money to buy a house or a car, you will pay a stream of equal payments.

**Future Value - annuity**

If you invest $1,000 each year at 8%, how much would you have after 3 years?

![Future Value Diagram](image)

**Calculator Solution:**

P/Y = 1  I = 8  N = 3
PMT = -1,000
FV = $3,246.40
Future Value - annuity
If you invest $1,000 each year at 8%, how much would you have after 3 years?

Calculator Solution:
P/Y = 1  I = 8  N = 3
PMT = -1,000
FV = $3,246.40

Future Value - annuity
If you invest $1,000 each year at 8%, how much would you have after 3 years?

Mathematical Solution:
FV = PMT (FVIFA \(_{i,a}\))
FV = 1,000 (FVIFA \(_{.08,3}\)) (use FVIFA table, or)

Future Value - annuity
If you invest $1,000 each year at 8%, how much would you have after 3 years?

Mathematical Solution:
FV = PMT (FVIFA \(_{i,a}\))
FV = 1,000 (FVIFA \(_{.08,3}\)) (use FVIFA table, or)
FV = PMT \(\left(\frac{(1+i)^n - 1}{i}\right)\)
### Future Value - annuity

If you invest $1,000 each year at 8%, how much would you have after 3 years?

Mathematical Solution:

\[
FV = PMT \times (FVIFA_{i,n})
\]

\[
FV = 1,000 \times (FVIFA_{0.08,3})
\]

(use FVIFA table, or)

\[
FV = PMT \times \frac{(1 + i)^n - 1}{i}
\]

\[
FV = 1,000 \times \frac{(1.08)^3 - 1}{0.08} = $3246.40
\]

### Present Value - annuity

What is the PV of $1,000 at the end of each of the next 3 years, if the opportunity cost is 8%?

### Calculator Solution:

\[
P/Y = 1 \quad I = 8 \quad N = 3
\]

\[
PMT = -1,000
\]

\[
PV = $2,577.10
\]
Mathematical Solution:

\[
PV = PMT \cdot PVIFA_{i,n}
\]

\[
PV = 1,000 \cdot PVIFA_{0.08,3}
\]

(use PVIFA table, or)

\[
PV = \frac{1}{i} \left[ \frac{1 - (1 + i)^n}{1 - (1 + i)^{n-1}} \right]
\]

\[
PV = \frac{1}{0.08} \left[ \frac{1 - (1.08)^3}{1 - (1.08)^{2}} \right] = 2,577.10
\]
Perpetuities

- Suppose you will receive a fixed payment every period (month, year, etc.) forever. This is an example of a perpetuity.
- You can think of a perpetuity as an annuity that goes on forever.

Present Value of a Perpetuity

- When we find the PV of an annuity, we think of the following relationship:

Mathematically,

\[
(PVIFA_{i, n}) = \frac{1}{i} \frac{1}{(1 + i)^n}
\]
Mathematically,

\[(PVIFA\ i,\ n) = \frac{1}{i} \left(1 - \frac{1}{(1+i)^n}\right)\]

We said that a perpetuity is an annuity where \(n = \infty\). What happens to this formula when \(n\) gets very, very large?

When \(n\) gets very large,

\[1 - \frac{1}{(1+i)^n} \rightarrow \frac{1}{i}\]

When \(n\) gets very large,

\[\frac{1}{(1+i)^n} \rightarrow 0\]

\[\frac{1}{i}\]

So we’re left with \(PVIFA = \frac{1}{i}\)

Present Value of a Perpetuity

- So, the PV of a perpetuity is very simple to find:

\[PV\ of\ a\ Perpetuity = \frac{1}{i}\]
Present Value of a Perpetuity

- So, the PV of a perpetuity is very simple to find:

\[ PV = \frac{PMT}{i} \]

What should you be willing to pay in order to receive $10,000 annually forever, if you require 8% per year on the investment?

\[ PV = \frac{10,000}{0.08} = \frac{10,000}{0.08} \]

= $125,000

Ordinary Annuity vs. Annuity Due
Begin Mode vs. End Mode

ordinary annuity

<table>
<thead>
<tr>
<th>Year</th>
<th>Begin Mode</th>
<th>End Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

PV in END Mode

FV in END Mode

PV in BEGIN Mode

FV in BEGIN Mode

Begin Mode vs. End Mode

annuity due

<table>
<thead>
<tr>
<th>Year</th>
<th>Begin Mode</th>
<th>End Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

PV in BEGIN Mode

FV in BEGIN Mode
Earlier, we examined this “ordinary” annuity:

Using an interest rate of 8%, we find that:

- The Future Value (at 3) is $3,246.40.
- The Present Value (at 0) is $2,577.10.

Earlier, we examined this “ordinary” annuity:

Using an interest rate of 8%, we find that:

- The Future Value (at 3) is $3,246.40.

What about this annuity?

• Same 3-year time line,
• Same 3 $1000 cash flows, but
• The cash flows occur at the beginning of each year, rather than at the end of each year.
• This is an “annuity due.”
Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

Calculator Solution:
Mode = BEGIN     P/Y = 1     I = 8
N = 3           PMT = -1,000
FV = $3,506.11

Mathematical Solution:
Simply compound the FV of the ordinary annuity one more period:

\[
FV = \text{PMT} \times (FVIFA_{i,n}) \times (1 + i)
\]

Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

Mathematical Solution: Simply compound the FV of the ordinary annuity one more period:

\[
FV = \text{PMT} \times (FVIFA_{i,n}) \times (1 + i)
\]

(Use FVIFA table, or)
Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

Mathematical Solution:
Simply compound the FV of the ordinary annuity one more period:
\[ FV = PMT \times (FVIFA_{i,n}) (1 + i) \]
\[ FV = 1,000 \times (FVIFA_{.08,3}) (1.08) \]
(1.08)3 = $3,506.11

Present Value - annuity due

What is the PV of $1,000 at the beginning of each of the next 3 years, if your opportunity cost is 8%?

Calculator Solution:
Mode = BEGIN     P/Y = 1     I = 8
N = 3     PMT =   1,000     PV =  $2,783.26

Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

Mathematical Solution:
Simply compound the FV of the ordinary annuity one more period:
\[ FV = PMT \times (FVIFA_{i,n}) (1 + i) \]
\[ FV = 1,000 \times (FVIFA_{.08,3}) (1.08) \]
(1.08)3 = $3,506.11

Present Value - annuity due

What is the PV of $1,000 at the beginning of each of the next 3 years, if your opportunity cost is 8%?

Mathematical Solution:
Present Value - annuity due

Mathematical Solution:
Simply compound the FV of the ordinary annuity one more period:

\[
P_{	ext{PV}} = PMT \cdot (PVIFA_{i,n}) \cdot (1+i)
\]

\[
P_{	ext{PV}} = 1,000 \cdot (PVIFA_{0.08,3}) \cdot (1.08)
\]

\[
P_{	ext{PV}} = 1,000 \cdot \frac{1 - (1 + 0.08)^{-3}}{0.08} \cdot (1.08) = \$2,783.26
\]
- Sorry! There’s no quickie for this one. We have to discount each cash flow back separately.

Uneven Cash Flows

<table>
<thead>
<tr>
<th>period</th>
<th>CF</th>
<th>PV (CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td>-10,000.00</td>
</tr>
<tr>
<td>1</td>
<td>2,000</td>
<td>1,818.18</td>
</tr>
<tr>
<td>2</td>
<td>4,000</td>
<td>3,305.79</td>
</tr>
<tr>
<td>3</td>
<td>6,000</td>
<td>4,507.89</td>
</tr>
<tr>
<td>4</td>
<td>7,000</td>
<td>4,781.09</td>
</tr>
</tbody>
</table>

PV of Cash Flow Stream: $4,412.95
Which is the better loan:
• 8% compounded annually, or
• 7.85% compounded quarterly?
We can’t compare these nominal (quoted) interest rates, because they don’t include the same number of compounding periods per year!
We need to calculate the APY.

\[
\text{APY} = \left(1 + \frac{\text{quoted rate}}{m}\right)^m - 1
\]

Find the APY for the quarterly loan:
\[
\text{APY} = \left(1 + \frac{0.0785}{4}\right)^4 - 1
\]
\[
\text{APY} = 0.0808, \text{ or } 8.08\%
\]
Annual Percentage Yield (APY)

\[ \text{APY} = \left( 1 + \frac{\text{quoted rate}}{m} \right)^m - 1 \]

- Find the APY for the quarterly loan:

\[ \text{APY} = \left( 1 + \frac{0.0785}{4} \right)^4 - 1 \]

\[ \text{APY} = 0.0808, \quad \text{or} \quad 8.08\% \]

- The quarterly loan is more expensive than the 8% loan with annual compounding!

Practice Problems

Example

- Cash flows from an investment are expected to be $40,000 per year at the end of years 4, 5, 6, 7, and 8. If you require a 20% rate of return, what is the PV of these cash flows?

How to solve:

1) Discount each cash flow back to time 0 separately.
How to solve:
1) Discount each cash flow back to time 0 separately.

Or,
2) Find the PV of the annuity:

PV: End mode; P/YR = 1; I = 20;
PMT = 40,000; N = 5
PV = \$119,624

Then discount this single sum back to time 0.

PV: End mode; P/YR = 1; I = 20;
N = 3; FV = 119,624;
Solve: PV = \$69,226

The PV of the cash flow stream is \$69,226.
Retirement Example

After graduation, you plan to invest $400 per month in the stock market. If you earn 12% per year on your stocks, how much will you have accumulated when you retire in 30 years?

Mathematical Solution:

Using your calculator:

\[
P/YR = 12 \\
N = 360 \\
PMT = -400 \\
I\%YR = 12 \\
FV = $1,397,985.65
\]
If you invest $400 at the end of each month for the next 30 years at 12%, how much would you have at the end of year 30?

Mathematical Solution:

\[ FV = PMT \times (FVIFA_{i,n}) \]

\[ FV = 400 \times (FVIFA_{.01,360}) \]  
(can’t use FVIFA table)

\[ FV = PMT \left( \frac{(1 + i)^n - 1}{i} \right) \]

\[ FV = 400 \left( \frac{(1.01)^{360} - 1}{.01} \right) = \$1,397,985.65 \]

If you borrow $100,000 at 7% fixed interest for 30 years in order to buy a house, what will be your monthly house payment?

House Payment Example:

\[ PMT = \frac{PV \times i}{1 - \left(1 + i\right)^{-n}} \]

\[ PMT = \frac{100,000 \times .07}{1 - \left(1 + .07\right)^{-30}} \]

\[ PMT = \$770.39 \]
House Payment Example

Mathematical Solution:

\[ PV = PMT \times (PVIFA_{i,n}) \]

100,000 = PMT \times (PVIFA_{0.07,360})  
(can’t use PVIFA table)

House Payment Example

Mathematical Solution:

\[ PV = PMT \left[ \frac{1}{i} - \frac{1}{(1+i)^n} \right] \]
**House Payment Example**

**Mathematical Solution:**

\[
PV = PMT \left( \frac{1}{PVIFA \, i, \, n} \right)
\]

\[
100,000 = PMT \left( PVIFA \, .07, \, 360 \right)
\]

(can't use PVIFA table)

\[
PV = PMT \left[ \frac{1}{(1 + i)^n} \right]
\]

\[
100,000 = PMT \left[ \frac{1}{1.005833^{360}} \right]
\]

\[
PMT = $665.30
\]

**Team Assignment**

Upon retirement, your goal is to spend 5 years traveling around the world. To travel in style will require $250,000 per year at the beginning of each year. If you plan to retire in 30 years, what are the equal monthly payments necessary to achieve this goal? The funds in your retirement account will compound at 10% annually.

- How much do we need to have by the end of year 30 to finance the trip?

\[
PV_{30} = PMT \left( PVIFA \, .10, \, 5 \right) \left( 1.10 \right) = 250,000 \left( 3.7908 \right) \left( 1.10 \right) = \$1,042,470
\]

- Now, assuming 10% annual compounding, what monthly payments will be required for you to have $1,042,466 at the end of year 30?

Using your calculator,

- **Mode = BEGIN**

  PMT = -$250,000

  N = 5

  I%YR = 10

  PV = $1,042,466

- **Mode = END**

  N = 360

  I%YR = 10

  P/YR = 12

  FV = $1,042,466

  PMT = -$461.17
• So, you would have to place $461.17 in your retirement account, which earns 10% annually, at the end of each of the next 360 months to finance the 5-year world tour.

\[ \text{Effective Annual Rates (EAR)} \]

**Effective Annual Rates (EAR) vs. Quoted Rate**

\[
\text{EAR} = \left[ 1 + \frac{\text{Quoted Rate}}{m} \right]^m - 1
\]

Q: You are offered 12 percent compounded monthly, what is the EAR?

\[ A: \text{ APR vs. EAR} \]

A: An APR of 18 percent with monthly payments is really \(.18/12 = .015\) or 1.5 percent per month.

\[
\text{Effective Annual Rates (EAR)}
\]

\[
\begin{align*}
\text{EAR} & = \left[ 1 + \frac{\text{Quoted rate}}{m} \right]^m - 1 \\
& = \left[ 1 + \frac{.12}{12} \right]^{12} - 1 \\
& = 1.01^{12} - 1 \\
& = 1.126825 - 1 \\
& = 12.6825% \\
\end{align*}
\]