Problem # 1

Maine Mountain Dairy claims that its eight-ounce low-fat yogurt cups contain, on average, fewer calories than the eight-ounce low-fat yogurt cups produced by a competitor. A consumer agency wanted to check this claim. A sample of 27 such yogurt cups produced by this company showed that they contained an average of 141 calories per cup. A sample of 25 such yogurt cups produced by its competitor showed that they contained an average of 144 calories per cup. Assume that the two populations are normally distributed with population standard deviations of 5.5 calories and 6.4 calories, respectively.

a. Make a 98% confidence interval for the difference between the mean number of calories in the eight-ounce low-fat yogurt cups produced by the two companies.
b. Test at the 1% significance level whether Maine Mountain Dairy’s claim is true.
c. Calculate the $p$-value for the test of part b. Based on this $p$-value, would you reject the null hypothesis if $\alpha = .005$? What if $\alpha = .025$?

Problem # 2

A car magazine is comparing the total repair costs incurred during the first three years on two sports cars, the T-999 and the XPY. Random samples of 45 T-999s and 51 XPYs are taken. All 96 cars are three years old and have similar mileages. The mean of repair costs for the 45 T-999 cars is $3300 for the first three years. For the 51 XPY cars, this mean is $3850. Assume that the standard deviations for the two populations are $800 and $1000, respectively.

a. Construct a 99% confidence interval for the difference between the two population means. Test the null hypothesis at the $\alpha = 0.10$ level of significance.
b. Using the 1% significance level, can you conclude that such mean repair costs are different for these two types of cars?
c. What would your decision be in part b if the probability of making a Type I error were zero? Explain.
Problem # 3

The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviations.

\[ n_1 = 25, \quad \bar{x}_1 = 12.50, \quad s_1 = 3.75 \]
\[ n_2 = 20, \quad \bar{x}_2 = 14.60, \quad s_2 = 3.10 \]

a. What is the point estimate of \( \mu_1 - \mu_2 \)?

b. Construct a 95% confidence interval for \( \mu_1 - \mu_2 \).

c. Test at the 5% significance level if the two population means are different.

Problem # 4

The management of a supermarket wanted to investigate whether the male customers spend less money on average than the female customers. A sample of 35 male customers who shopped at this supermarket showed that they spent an average of $80 with a standard deviation of $17.50. Another sample of 40 female customers who shopped at the same supermarket showed that they spent an average of $96 with a standard deviation of $14.40.

a. Construct a 99% confidence interval for the difference between the mean amounts spent by all male and all female customers at this supermarket.

b. Using the 2.5% significance level, can you conclude that the mean amount spent by all male customers at this supermarket is less than that spent by all female customers?

Problem # 5

An insurance company wants to know if the average speed at which men drive cars is higher than that of women drivers. The company took a random sample of 27 cars driven by men on a highway and found the mean speed to be 72 miles per hour with a standard deviation of 2.2 miles per hour. Another sample of 18 cars driven by women on the same highway gave a mean speed of 68 miles per hour with a standard deviation of 2.5 miles per hour. Assume that the speeds at which all men and all women drive cars on this highway are both normally distributed with unequal population standard deviations.

a. Construct a 98% confidence interval for the difference between the mean speeds of cars driven by all men and all women on this highway.
b. Test at the 1% significance level whether the mean speed of cars driven by all men drivers on this highway is higher than that of cars driven by all women drivers.

**Problem # 6**

A private agency claims that the crash course it offers significantly increases the writing speed of secretaries. The following table gives the scores of eight secretaries before and after they attended this course.

<table>
<thead>
<tr>
<th>Before</th>
<th>81</th>
<th>75</th>
<th>89</th>
<th>91</th>
<th>65</th>
<th>70</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>97</td>
<td>72</td>
<td>93</td>
<td>110</td>
<td>78</td>
<td>69</td>
<td>115</td>
</tr>
</tbody>
</table>

a. Make a 90% confidence interval for the mean $\mu_d$ of the population paired differences, where a paired difference is equal to the score before attending the course minus the score after attending the course

b. Using the 5% significance level, can you conclude that attending this course increases the writing speed of secretaries?

**Problem # 7**

Find the following confidence intervals for $\mu_d$ assuming that the populations of paired differences are normally distributed.

a. $n = 12, d = 17.5, s_d = 6.3$, confidence level = 99%.

b. $n = 27, d = 55.9, s_d = 14.7$, confidence level = 95%.

c. $n = 16, d = 29.3, s_d = 8.3$, confidence level = 90%

**Problem # 8**

A sample of 1000 observations taken from the first population gave $x_1 = 290$. Another sample of 1200 observations taken from the second population gave $x_2 = 396$.

a. Find the point estimate of $p_1 - p_2$.

b. Make a 98% confidence interval for $p_1 - p_2$.

c. Show the rejection and nonrejection regions on the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for $H_0$: $p_1 = p_2$ versus $H_1$: $p_1 < p_2$. Use a significance level of 1%.
d. Will you reject the null hypothesis mentioned in part c at a significance level of 1%?

**Problem # 9**

Conduct the following tests of hypotheses assuming that the populations of paired differences are normally distributed.

a. $H_0: \mu_d = 0, H_1: \mu_d \neq 0, n = 26, d = 9.6, s_d = 3.9, \alpha = .05$

b. $H_0: \mu_d = 0, H_1: \mu_d > 0, n = 15, d = 8.8, s_d = 4.7, \alpha = .01$

c. $H_0: \mu_d = 0, H_1: \mu_d < 0, n = 20, d = -7.4, s_d = 2.3, \alpha = .10$

**Problem # 10**

Primary seat belt laws allow police to stop a motorist for not wearing a seat belt. According to the National Highway Traffic Safety Administration, the percentage of drivers wearing seat belts in the state of Tennessee was 68.5% in 2003 and 72% in 2004 (*USA TODAY*, November 23, 2004). (Tennessee passed a primary seat belt law in July 2004.) Assume that these percentages are based on random samples of 1000 drivers in 2003 and 1050 in 2004.

a. Construct a 98% confidence interval for the difference between the two population proportions.

b. Test at the 1% significance level whether the proportion of all drivers in Tennessee who used seat belts in 2003 is lower than that for 2004. Use the critical-value approach.

c. Repeat part b using the p-value approach.