Problem # 1

Consider the following contingency table that records the results obtained for four samples of fixed sizes selected from four populations.

<table>
<thead>
<tr>
<th>Sample Selected From</th>
<th>Population 1</th>
<th>Population 2</th>
<th>Population 3</th>
<th>Population 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>24</td>
<td>81</td>
<td>60</td>
<td>121</td>
</tr>
<tr>
<td>Row 2</td>
<td>46</td>
<td>64</td>
<td>91</td>
<td>72</td>
</tr>
<tr>
<td>Row 3</td>
<td>20</td>
<td>37</td>
<td>105</td>
<td>93</td>
</tr>
</tbody>
</table>

a. Write the null and alternative hypotheses for a test of homogeneity for this table.
b. Calculate the expected frequencies for all cells assuming that the null hypothesis is true.
c. For $\alpha = .025$, find the critical value of $\chi^2$. Show the rejection and nonrejection regions on the chi-square distribution curve.
d. Find the value of the test statistic $\chi^2$.
e. Using $\alpha = .025$, would you reject the null hypothesis?

Problem # 2

A forestry official is comparing the causes of forest fires in two regions, A and B. The following table shows the causes of fire for 76 randomly selected recent fires in these two regions

<table>
<thead>
<tr>
<th>Region</th>
<th>Arson</th>
<th>Accident</th>
<th>Lightning</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region A</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Region B</td>
<td>7</td>
<td>14</td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

Test at the 5% significance level whether causes of fire and regions of fires are related.

Problem # 3

A sample of 25 observations selected from a normally distributed population produced a sample variance of 35. Construct a confidence interval for $\sigma^2$ for each of the following confidence levels and comment on what happens to the confidence interval of $\sigma^2$ when the confidence level decreases

a. $1 - \alpha = .99$
b. $1 - \alpha = .95$
c. $1 - \alpha = .90$
Problem # 4

Professor Fox’s “50-minute” lectures vary in length. Professor Fox claims that the variance of the lengths of his lectures is within 2 square minutes. A random sample of 23 of these lectures was timed, and the variance of the lengths of these lectures was found to be 2.7 square minutes. Assume that the lengths of all such lectures by Professor Fox are (approximately) normally distributed.

a. Make the 98% confidence intervals for the variance and standard deviation of the lengths of all 50-minute lectures by Professor Fox.
b. Test at the 1% significance level whether the variance of the lengths of all such lectures by Professor Fox exceeds 2 square minutes.

Problem # 5

The manufacturer of a certain brand of light bulbs claims that the variance of the lives of these bulbs is 4200 square hours. A consumer agency took a random sample of 25 such bulbs and tested them. The variance of the lives of these bulbs was found to be 5200 square hours. Assume that the lives of all such bulbs are (approximately) normally distributed.

a. Make the 99% confidence intervals for the variance and standard deviation of the lives of all such bulbs.
b. Test at the 5% significance level whether the variance of such bulbs is different from 4200 square hours